

LIMITS FOR THE GENERATION OF ULTRA-SHORT UNDAMPED ELECTRICAL WAVES

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The output characteristics of a generator for undamped electrical waves, may be represented by the following equation:-

$$N = \frac{\bar{U} \bar{i}}{\eta_E} - \frac{G_1}{\sqrt{\lambda}} \hat{U}_2 \quad \text{..... (1)}$$

where N is the useful output

\bar{U} is the input d.c. voltage

\bar{i} is the mean value of d.c. input

η_E is the efficiency of electron-flow

\hat{U} is the peak alternating voltage of the excitation circuit

G_1 is the loss factor (for a wavelength of $\lambda = 1$ cm) for the oscillating system, comprising the exciter-oscillator feed-back and control-circuitry.

For a given wavelength λ , the useful output N is thus dependent upon a number of parameters, i.e. \bar{U} , \bar{i} and η_E , the physical dimensions of the construction, and upon constants inherent in the materials used, such as the specific resistance of the H. F. conductors. Fundamentally these parameters must be such that N can assume its maximum possible value N_{\max} for any given wavelength. It is important to know how N_{\max} decreases with respect to the wavelength. Evidently there must be an ultra short wavelength, which it is just possible to produce for which N_{\max} would be exactly nil.

One ought not to expect to find a complete solution of this problem within the scope of this paper; what I believe I can furnish is a theoretical treatment of the subject, carried through to the point of a numerical evaluation but this only on a very much simplified example. If, however, some factors should emerge which appear useful for the purpose of a systematic advance into the field of very short undamped waves, then this will have been made possible because all the considerations have been based on the mechanics of the electron and Maxwell's theory. Thus calculation becomes possible and sufficient expert opinion is available regarding the significance of certain omissions, and the degree of approximation to real conditions.

Let us assume that the effect of space-charges can be neglected, and that there are furthermore no other limitations to the increase of the mean direct current \bar{i} , then referring to equation (1) and for any given wavelength λ , any magnitude of useful output N will be attainable, this with a higher overall efficiency

$$\eta = \frac{N}{\bar{U} \bar{i}} = \eta_E \left[1 + \frac{\frac{G_1}{\sqrt{\lambda}} \hat{U}^2}{N} \right]^{-1} \quad \text{..... (2)}$$

if the efficiency of electron-flow η_E is increased and the smaller the loss in the oscillatory system (which is proportional to \hat{U}^2) relative to the useful output N . Having once arrived (by virtue of suitably chosen parameters) at the most advantageous design for the generator from the efficiency point of view for a given wavelength, then corresponding designs may similarly be drawn up for any other wavelength, since in accordance with the laws of similarity of the

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mechanics of the electron in conjunction with Maxwell's theory, η_E and G_1 remain invariable, when all linear dimensions are changed in proportion to the wavelength λ and all voltages \bar{U} , \hat{U} , etc. are kept constant. If static magnetic fields are involved in the transport of electrons, these should be changed in the ratio of $1/\lambda$, in any transformation process.

In accordance with equation (2), therefore, η decreases with λ , but the attainable useful output N nevertheless remains unlimited, until the field strengths (increasing with $1/\lambda$) at the critical point of the conductor surfaces reach the value at which interfering electronic self-discharges occur. It is from this wavelength downwards that the generator will fail. If above the limit set to this ultra short wavelength by electronic self-discharge, there are further limitations to the attainment of any desired value of useful output N , such limitation must be attributed to a factor so far disregarded namely the limits of \bar{i} . Each of these limits which now warrant further consideration permits the derivation of an equation for \bar{i} which in conjunction with equation (1) enables the maximum useful output N_{\max} for any given wavelength λ to be derived theoretically and evaluated numerically.

Heat dissipation requires that

$$\bar{i} = \frac{k \lambda^2 - \frac{1}{2} \frac{G_1}{\sqrt{\lambda}} \hat{U}^2}{\bar{U} (1 - \eta_E)} \quad \dots\dots (3)$$

where k is a constant, characteristic of the intensity of cooling and numerically determined by the dimensions of the generator.

If for example the current density at the cathode surface cannot be increased beyond a certain value, we have

$$\bar{i} = \bar{i}_1 \lambda^2, \quad \dots\dots (4)$$

where \bar{i}_1 is a constant, characteristic of maximum current density.

If it is admitted that space-charges do in fact exert a certain influence, it follows from the field laws of electrostatics that this influence would be unaffected by the above-mentioned transformation of the generator construction, if the total current strength

$$\bar{i} = \bar{i}_0 \quad \dots\dots (5)$$

remains constant.

In order to be able to substitute definite values for k , \bar{i}_1 and \bar{i}_0 respectively, which depend upon the parameters of the generator construction, and so to arrive at numerical results, the following simple assumptions are made:-

- (1) The excitation electron flow from cathode to anode is in the form of a parallel beam of circular cross section d .
- (2) The control is in the immediate vicinity of the cathode and is wattless.
- (3) The controlled excitation electron flow consists of electron bunches or pulses of very short duration which succeed each other at time intervals of one h. f. cycle; these pulses have a mean d.c. value of \bar{i} and uniform velocity \bar{U} .
- (4) A steady field does not exist over the excitation path d .
- (5) The excitation resonator is a cylindrical, concentric cavity-resonator, namely a compromise (having the lowest possible losses) between the "narrow tube" and "flat box" types as shown in Figure 2b of reference [1]*.

* For references see end.

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The concentrated capacitor incorporated in the inner conductor has the capacitance:

$$C = \frac{q/\lambda^2}{d/\lambda} \quad \text{..... (6)}$$

where q is the cross section of the parallel electron beam and d is the excitation path.

The generator calculation proceeds as follows:

- (a) A suitable value is chosen for $\zeta_a = 2\pi r_a/\lambda$ i.e. $\zeta_a = 1.6$. The report referred to above [1] provides both for the "narrow tube" and "flat box" respectively and for a suitably chosen series of values of the parameter g a ζ_i value for which the relationship $G = G_i/\sqrt{\lambda}$ will be a minimum. Because of the requirement for resonance, each ζ_i value possesses a corresponding α value. If the minimum $G_{i\min}$ and the related values of $\zeta_{i\min}$ and α_{\min} are plotted against the parameter g , it is found in the case of both the "narrow tube" and the "flat box" that as the capacitance g increases, the loss factor $G_{i\min}$ and the reduced inner radius $\zeta_{i\min}$ both increase monotonically, whereas the reduced axial length α_{\min} decreases monotonically. The same applies to the above-mentioned "intermediate form" which is of interest here (see Figure 2b of "Cavity report" [1]) and for the former and the assumed value for ζ_a , $G_{i\min}$, $\zeta_{i\min}$ and α_{\min} should be equal to the arithmetic mean of the corresponding values of "narrow tube" and "flat box" respectively. The terms $G_{i\min}$, $\zeta_{i\min}$ and α_{\min} for the assumed value of ζ_a which depend only on g may be represented in terms of a finite powers series in g and can be interpolated.

- (b) Taking the surface area of the exciter resonator as cooling surface we have from (a) for equation (3)

$$k = \frac{1}{2\pi} \zeta_a (\zeta_a + \alpha_{\min}) (\theta_w - \theta_f) u. \quad \text{..... (7)}$$

where $(\theta_w - \theta_f)$ is the permissible temperature difference between the wall and the cooling medium and u is the known heat-transfer coefficient in watts per cm^2 per degree.

- (c) If j is maximum permissible current density in the process of emission from the cathode then for equation (4) we have:

$$\bar{i}_1 = j q/\lambda^2. \quad \text{..... (8)}$$

- (d) The limit \bar{i}_0 for \bar{i} in equation (5) imposed by the effect of space charges is

$$\bar{i}_0 = \frac{10^{-3}}{\pi c^2} \sqrt{\frac{2e}{m}} \bar{U}^{3/2} \cdot \frac{\Delta s}{s} \frac{q/\lambda^2}{(D/\lambda)^2} \quad \text{..... (9)}$$

where m and e are the mass and charge of the electron respectively, D is the transit path of the electron bunches between the point of origin and the centre of the excitation path, $\Delta s/s$ is the increase in relative "bunch length" in the direction of transit, due to the repulsion of the space-charge. In order to obtain the highest possible value for \bar{i}_0 , D/λ must be made as small as possible compatible with design requirements. A value for $\Delta s/s$ must be assumed. Excessive values mean indistinct pulses which affect the efficiency η_E .

- (e) If the ratio \hat{U}/\bar{U} is suitably chosen, the electron-bunches will arrive at the anode with zero velocity and the efficiency of electron-flow becomes $\eta_E = 1$. According to unpublished calculations by Kleinstaubner the ratio \hat{U}/\bar{U} which would satisfy this condition is the function of a single variable namely that of the parameter

$$\beta = \frac{m (2\pi c)^2}{e \bar{U}} \left[\frac{d}{\lambda} \right]^2 \quad \text{..... (10)}$$

This dependency on β is used in order to express \hat{U} in terms of \bar{U} in equation (1).

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- (f) If in equation (1) we now substitute consecutively \bar{l} from equations (3), (4) and (5), taking into account equations (7), (8) and (9), and that in accordance with (a) G_1 is determined by parameter g (equation (6)), we obtain one equation corresponding to equation (1) for each of the following effects: heat dissipation, current density, and space charge, in which the only remaining important parameters to be determined are \bar{U} , η/λ^2 and d/λ . The discussion of these three equations will supply numerically the fundamentals for the best choice of the remaining parameters and thus one function N_{\max} of λ in each case. For the wavelength concerned the smallest of the three N_{\max} values is decisive, only as long as the wavelength remains above the limiting value below which electronic self discharges occur.

Similar considerations, as shown in the above-mentioned simple example may be applied to other types of excitation systems and resonators.

REFERENCES

- [1] W. DÄLLENBACH. "Conditions for resonance oscillating field energy, power-loss, damping-coefficient and frequency variation of circular cylindrical-concentric cavity resonators". Hochfrequenztechnik u. Elektroskustik **61** (1943) 129-140.